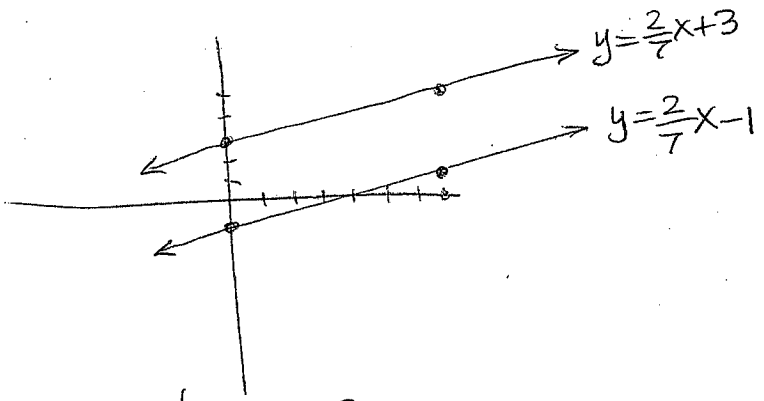
Math 45 SSM 2/e 3.6 Parallel and Perpendicular Lines

Objectives: 1) Determine if two lines (whose equations are given) are parallel.
2) Ditto, perpendicular.
3) Write equation of a new line parallel to a given line.
4) Ditto, perpendicular.

Explore

① Graph $y=\frac{2}{7}x-1$



② On the same axes, graph $y=\frac{2}{7}x+3$

⇒ what can we conclude about the graphs?
The lines are parallel; they never intersect.

⇒ what can we conclude about the equations?
They have the same slopes but different y-intercepts.

PARALLEL LINES
- never intersect
- have the same slope
- have different y-intercepts

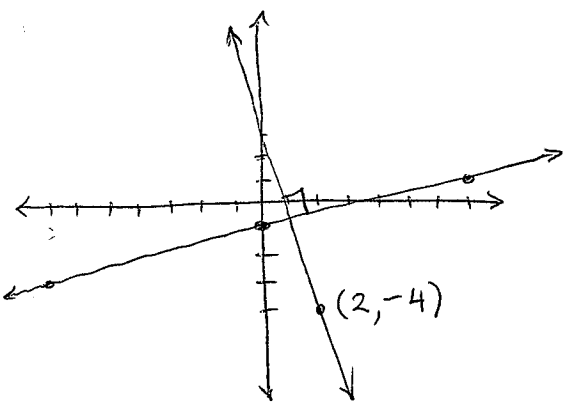
③ New axes, graph $y=\frac{2}{7}x-1$ again.

④ Plot $(2,-4)$

⑤ Free-hand; try to draw a line perpendicular to $y=\frac{2}{7}x-1$ that passes through $(2,-4)$

⑥ What is the slope of the line perpendicular?

$\boxed{\frac{-7}{2}}$ opposite of $\frac{2}{7}$ and reciprocal of $\frac{2}{7}$



⑦ Write the equation of the perpendicular line.

step 1: point-slope formula

$y - y_1 = m(x - x_1)$

step 2: slope $m = -\frac{7}{2}$ (opposite and reciprocal)

point $(x_1, y_1) = (2, -4)$

$y + 4 = -\frac{7}{2}(x - 2)$

step 3: $y + 4 = -\frac{7}{2}x + 7$ distribute

$-4 \qquad -4$

$$\boxed{y = -\frac{7}{2}x + 3}$$

Notice: The y-intercept of the new line is 3.
This is different from the y-int of the original line.
It's also different from the y coordinate of the point (2, −4).
There are no shortcuts to write eqn of line.

⑧ Write the equation of a line parallel to $y = \frac{2}{7}x - 1$ passing through (2, −4).
(Same line, same point)

step 1: point-slope formula

$y - y_1 = m(x - x_1)$

step 2: same slope

$m = \frac{2}{7}$

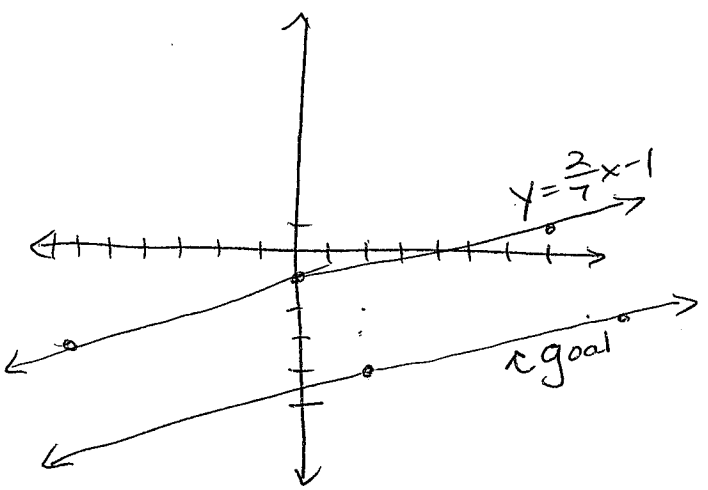
point $(x_1, y_1) = (2, -4)$

$y + 4 = \frac{2}{7}(x - 2)$

step 3: $y + 4 = \frac{2}{7}x - \frac{4}{7}$

$y = \frac{2}{7}x - \frac{4}{7} - \frac{4}{1}\cdot\frac{7}{7}$

$$\boxed{y = \frac{2}{7}x - \frac{32}{7}}$$



$\frac{32}{7} \approx 4.6$ matches sketch ☺

PERPENDICULAR LINES

- intersect to form a right angle
- have slopes which are both opposites & reciprocals.

(y-intercepts irrelevant)

To determine if two lines are parallel:
step 1: calculate slopes of each (usually by writing in $y=mx+b$ form)
step 2: if both slopes are the same number, yes, parallel.
if slopes are different in any way (sign, number), no, not parallel.
step 3: if y-ints and slopes are the same, same line, no, not parallel.

To determine if two lines are perpendicular:
step 1: calculate slopes of each (usually by writing in $y=mx+b$ form).
step 2: are the slopes opposite? if yes, go to step 3.
if no, write no, not perpendicular
are the slopes reciprocals? if yes, perpendicular.
if no, not perpendicular.

Usually these two processes are combined into one type of question:
Determine if the two lines are parallel, perpendicular, or neither.

(9) $-3x+y=5$ ← L_1
$6x-2y=-2$ ← L_2

step 1: $-3x+y=5$
$+3x \quad +3x$
$y=3x+5$ L_1 has slope 3.

$6x-2y=-2$
$-6x \quad -6x$
$\frac{-2y}{-2}=\frac{-6x-2}{-2}$
$y=3x+1$ L_2 has slope 3.

step 2: L_1 and L_2 have the same slope, $m=3$, but different y-ints.

L_1 and L_2 are parallel

(10) $2x+3y=-6$ ← L_1
$3x-2y=2$ ← L_2

step 1: $2x+3y=-6$
$-2x \quad -2x$

cont →

(12) cont. $\frac{3y}{3} = \frac{-2x}{3} + \frac{6}{3}$

$y = -\frac{2}{3}x + 2$ L_1 has slope $-\frac{2}{3}$.

$3x - 2y = 2$

$-3x \qquad -3x$

$\frac{-2y}{-2} = \frac{-3x}{-2} + \frac{2}{-2}$

$y = \frac{3}{2}x - 1$ L_2 has slope $\frac{3}{2}$

step 2: L_1 and L_2 have slopes which are opposites and reciprocals. $-\frac{2}{3} \leftrightarrow \frac{3}{2}$

L_1 and L_2 are perpendicular

CAUTION: Parallel and perpendicular are concepts that require two lines (two slope values). A single line by itself cannot be parallel or perpendicular to itself.

(11) $x + y = 7$ $\leftarrow L_1$

$x - y = 6$ $\leftarrow L_2$

step 1: $x + y = 7$

$y = -x + 7$ $m = -1$ is slope of L_1

$x - y = 6$

$-y = -x + 6$

$y = x - 6$ $m = 1$ is slope of L_2.

step 2: slopes are opposites.

what is the reciprocal of 1? $\frac{1}{1} = 1$.

So they are reciprocals also!

L_1 and L_2 are perpendicular

(12) Write the equation of a line perpendicular to $2x+y=5$ which passes through $(-1,3)$ in slope-intercept form.

NOTE: "passes through" or just "through" or "contains" all mean that the given point is a point on the line whose equation you are writing.

Step 1: Find the slope of L_1 whose equation is given.

$$2x+y=5$$
$$-2x \quad -2x$$
$$y=-2x+5 \qquad m=-2.$$

Step 2: Perpendicular means opposite reciprocal slope. Take the opposite and reciprocal of the slope in step 1.

opposite $\Rightarrow 2$ { reciprocal $\Rightarrow -\frac{1}{2}$
reciprocal $\Rightarrow \frac{1}{2}$ { opposite $\Rightarrow \frac{1}{2}$

NOTE: It doesn't matter which you do first, but you must do both

Step 3: Use the point-slope formula, the slope from step 2, and the point given in the problem.

$$y-3=\tfrac{1}{2}(x+1)$$
$$y=\tfrac{1}{2}x+\tfrac{1}{2}+3 \quad \cdot\tfrac{2}{2}$$
$$\boxed{y=\tfrac{1}{2}x+\tfrac{7}{2}}$$

Math 45 SSM 2/e 3.6 p. 16

(13) Write the equation of the line parallel to $2x+y=5$ which passes through $(-1,3)$ in slope-intercept form.

NOTE: "In slope-intercept form" refers to the final answer, not necessarily the method.

step 1: Find the slope of L_1 whose equation is given.

$$2x+y=5$$
$$-2x \quad -2x$$
$$y=-2x+5 \qquad m=-2.$$

step 2: Parallel means same slope. No calculations needed.

step 3: Use the point-slope formula, the slope from step 1, and the point given in the problem.

$$y-3=-2(x+1)$$
$$y=-2x-2+3$$
$$\boxed{y=-2x+1}$$

Extra practice

Parallel, perpendicular, or neither?

(14) $\left.\begin{array}{l} x-2y=-8 \\ x+2y=2 \end{array}\right\}$ $\leftarrow L_1$, $\leftarrow L_2$

$$x-2y=-8$$
$$-x \qquad -x$$
$$\frac{-2y}{-2}=\frac{-x}{-2}-\frac{8}{-2}$$
$$y=\tfrac{1}{2}x+4 \qquad m=\tfrac{1}{2} \text{ for } L_1.$$

$$x+2y=2$$
$$-x \qquad -x$$
$$\frac{2y}{2}=\frac{-x}{2}+\frac{2}{2}$$
$$y=-\tfrac{1}{2}x+1 \qquad m=-\tfrac{1}{2} \text{ for } L_2$$

$\frac{1}{2} \neq -\frac{1}{2} \Rightarrow$ not parallel

but $\left.\begin{array}{l} \frac{1}{2} \Rightarrow -2 \\ -\frac{1}{2} \Rightarrow +2 \end{array}\right\}$ would be opposites and reciprocals.

$\Rightarrow$ not perpendicular

$\boxed{\text{neither}}$

(15) Write the equation of a line perpendicular to $x+4y=2$ through $(-7,2)$ in slope-intercept form.

step 1: Find the slope of $x+4y=2$

$-x \quad -x$

$\frac{4y}{4} = \frac{-x}{4} + \frac{2}{4}$

$y = -\frac{1}{4}x + \frac{1}{2}$ $\quad m = \underline{\underline{-\frac{1}{4}}}$

step 2: perpendicular means opposite + reciprocal slope:

opposite $\frac{1}{4}$ $\quad$ reciprocal $\frac{4}{1} = 4$

reciprocal $\frac{4}{1} = 4$ $\quad$ opposite 4

step 3: $y - y_1 = m(x - x_1)$

$y - 2 = 4(x+7)$

$y - 2 = 4x + 28$

$+2 \qquad +2$

$\boxed{y = 4x + 30}$

(16) Write the equation of a line parallel to $x+4y=2$ through $(-7,2)$ in slope-intercept form.

step 1: Same as (6) above. $m = -\frac{1}{4}$

step 2: use same slope.

step 3: $y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{4}(x+7)$

$y - 2 = -\frac{1}{4}x - \frac{7}{4}$

$+2 \qquad +2 \cdot \frac{4}{4}$

$\boxed{y = -\frac{1}{4}x + \frac{1}{4}}$

CAUTION: The y-intercept of the answer almost always has nothing to do with the y-intercept of the given line. You cannot shortcut step 3.